# Finite and Infinite Sets

The set of all elements of the universal set U which are not in A, is called the complement of the set A.

$$A' = \{x | x \in U \text{ and } x \notin A\}$$

Consider the set A = {3, 5, 7, 9, 11} that is the number of elements in A is definite.

Now, consider the set B = $\{x|x$ is a natural number greater than 7 and less than 15}. In order to find the number of elements in the set B, let us write the set in the list form. Thus, B = {8, 9, 10, 11, 12, 13, 14}

It is easy to see that there are seven elements in B and 7 is also a definite number. Sets like A and B are finite sets. A set is called a finite set if the number of elements in the set is definite.

If A is a finite set, $n(A)$ denotes the number of elements in A.

In the above-mentioned sets, $n(A) = 5$ and $n(B) = 7$

There are no elements in the null set. So the number of elements is 0, which is a definite number. Thus, the null set is a finite set.

Now, let us consider the set N of natural numbers, that is,

$$N = \{x | x \text{ is a natural number}\}$$

If we express this set in the list form, N = $\{1, 2, 3, 4, 5, \ldots\}$ the list is unending because the number of elements of N is not definite.

∴ N is not a finite set. Such sets are infinite.

# Introduction to Sets

## Objectives

In this session we will understand the concept of a set and learn the terms associated with a set.

Dozo and Mozo are in a convenience store. They observe that the articles on the shelves are arranged in a particular manner. There are display plates hanging from the top. Dozo asks Mozo to take note of the different categories mentioned: Cosmetics, Toiletries, Stationery, Confectionery, Garments, Electronics, Music, Furniture, Fruits and vegetables, Beverages, etc.

All these groups are made for easy identification and location of things you want to buy. In certain situations, a particular item may find place in two different categories. For example, an item like shampoo can be placed in the category of toiletries or cosmetics.

We have a similar but well defined concept in Mathematics wherein we call these groups as sets. Let us define a set precisely.

Consider some geometrical figures to understand the concept of a set. We want to have a set of polygons. A set is defined as a collection of well-defined, distinct objects. The objects are called elements or members of the set. In this case, the geometrical figures are the elements of the collection.

The important words in the definition are distinct and well-defined. By distinct, we mean that an element appears only once in the set. We observe that the $5^{th}$ and $6^{th}$ elements are not distinct; both are hexagons. Thus, one of the Hexagons has to be removed from the collection.

Well-defined implies the absence of ambiguity that is we should be able to decide clearly if an object is a member of the set or not.

This collection contains a circle, which is not a polygon and hence has to be dropped from the collection. Now, we see that the collection contains only polygons and polygon appears only once. We can also say that all the elements are satisfying the condition of belonging to the same class that is they are polygons.

The elements of a set are written in curly parenthesis separated by commas. The sets are generally named using capital letters and we use lower case letters for elements.

## Set Representation

We can represent a set in two ways. The first one is called Listing or Roster method. In this case, all the elements of the set are written within the curly brackets separated by commas. For example, the set B of natural numbers between 2 and 11 is written as B = {3, 4, 5, 6, 7, 8, 9, 10}. A set C of alphabets from 'a' to 'f' is written in Roster form as C = {a, b, c, d, e, f}.

The second method of writing a set is called Property or Set-builder Notation. In this case, the set is expressed by the common characteristic property that the elements of the set must possess. For example, B = {3, 4, 5, 6, 7, 8, 9, 10} can be written in set-builder form as $B = \{x \mid x \in N \text{ and } 2 < x < 11\}$.The same set can also be written as $B = \{x \mid x \in N \text{ and } 3 \leq x \leq 10\}$

We can express the same set by formulating a different rule, as long as it has the same elements. For example: $B = \{x \mid x \in N \text{ and } 2< x \leq 10\}$ or $B = \{x \mid x \in N \text{ and } 3 \leq x < 11\}$ or $B = \{x \mid x \in Z \text{ and } 3 \leq x \leq 10\}$ etc.

Thus, we conclude that a set can be written in many ways in Set- builder Notation.

## Summary

In this session we have understood the concept and ways of writing a

## Methods of Describing a Set

There are basically two methods of describing a set:

(1) Listing method: listing its members or elements

(2) Property method: describing the properties of its members

### The listing method

In this method, all members of the set are written in brackets with commas between two consecutive members.

Now, if we want to consider the set of all natural numbers from 1 to 1000, the list will be too long to write. So we write a few members in the beginning, followed by three dots and finally the last number. Thus, the set of all natural numbers from 1 to 1000 is usually written as

$\{1, 2, 3, . \dots, 1000\}$

The following two points should be remembered about this method.

(i) Only the distinct members are listed. For example, the set of letters of the word MATHEMATICS is listed as {M, A, T, H, E, I, C, S}. Though the letters M, A, T are repeated in the spelling, they are listed only once.

(ii) The order in which the members of the set are written is immaterial.

### Property method

In this method, a symbol like $x$ is used for describing the set.

Suppose, B = $\{1, 2, 3, 4, 5, 6\}$.

Each member of B is a positive integer less than 7. So, according to this method, we write

B = $\{x | x \text{ is a positive integer and } x < 7\}$

This may be read as “B is the set of members $x$ such that $x$ is a positive integer and $x$ is less than 7”. Thus, $x$ is a representative element of the set B. Instead of the letter $x$, symbols like a, b, m, n can also be used.

While describing the properties of set B, after writing $x$ in the brackets, in place of the vertical line we may put a pair of dots.

## Notations for a Set

Brackets, floral braces are used to denote a set and the elements of the set are written in the brackets. Thus, the set whose members are 2, 4, 6, 8 is written as $\{2, 4, 6, 8\}$

If we call this set A, then we can write A = $\{2, 4, 6, 8\}$

The symbol $\in$ is used to express the phrase that ‘an element belongs to a set’. Since 4 is an element of set A, we write ‘4 $\in$ A’ and read this as ‘4 is an element of set A’, ‘4 $\in$ A’ or simply ‘4 is in set A’.

The symbol $\notin$ is used to express the fact that an object is not a member of a set. For example, 5 is not a member of A; so, we write ‘5 $\notin$ A’ and read this as ‘5 does not belong to A’ or ‘5 is not in set A’.

## One to One Correspondence and Equivalent Sets

Suppose the members of the family are dinning and plates are prepared on the table then there is a One to one correspondence between the set of family members and the plates prepared.

We will indicate this correspondence by the symbol $\leftrightarrow$. If each element of a set A is associated with one and only one element of a set B and each element of B is associated with one and only one element of set A, then the sets A and B are said to be in one to one correspondence.

If both the sets are finite and if they are in one to one correspondence, it is obvious that they have the same number of elements.

If two sets have the same number of elements, they are said to be equivalent sets. Using symbol, we write $A \sim B$. Between two equivalent sets, a one to one correspondence can be established.

# Power Set of a Set

**Objective**

In this session, we will learn the concept of the Power Set of a set.

Let us consider a few sets to understand the concept. Consider a set A = {1, 2} of cardinality 2. Let us list all the possible subsets of set A.

The subsets of A are { }, {1}, {2}, {1, 2}. So, the set of all the subsets of A = { { }, {1}, {2}, {1, 2} }.

Let us consider another set S = {a, b, c} of cardinality 3. The subsets of S are: { }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}.

The set of all the subsets of S is { { }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c} }.

This leads us to define a special set called Power Set. The Power Set of set X is the set of all the subsets of set X and is written as P(X). Thus, the Power set of A = { { }, {1}, {2}, {1, 2} } and Power set of S = {{ }, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c} }

Observe that n(A) = 2 and n(P(A)) = 4. In the second case, we have n (S) = 3 and n(P(S)) = 8

From the above illustrations, we observe that when n(A) = 2, n(P(A)) = 4 = $2^2$ and when n(S) = 3, n(P(S)) = 8 = $2^3$

In general, for a finite set with cardinality 'n', the cardinality of the power set is $2^n$. That is, a set with n elements has $2^n$ subsets.

We notice that the elements of the Power set of a set are sets and not individual elements.

For example, for A = {1, 2}

Here, $1 \notin P(A)$ but $\{1\} \in P(A)$; also, $\{1\} \not\subset P(A)$ but $\{\{1\}\} \subset P(A)$

**Summary**

In this session we have learnt the concept of the Power Set of a set and also learnt that the elements of a Power set are sets and not the individual elements. We have also learnt to find the cardinal number of the power set when the cardinal number of the set is known.

## Power Set

Let A be = {a, b}. The subsets of A are $\emptyset$, {a}, {b}, {a, b}. The set of subsets is called the power set of the set A. The power set of a set is the set of all the subsets of the given set. Here, the members of a power set are themselves sets. We denote the power set of A by P (A). Every subset of A is a member of P(A). Hence, $P(A) = \{B \mid B \subset A\}$

For every set A, $\emptyset \subset A \Rightarrow \emptyset \in P(A)$ and $\{a\} \subset A \Rightarrow \{a\} \in P(A)$. Thus, $P(A) \neq \emptyset$

Again, the set A containing n elements has $2^n$ subsets. So, it is clear that the number of elements in the power set of A containing n elements is $2^n$.

# Set Theory: Introduction

## Representation of a set

Any set can be represented by one or both of the following methods: 1. Listing Method 2. Property Method

In listing method, all the members or elements of the set are enlisted within the braces. For example: A = {a, b, c, d}. Here, A is the set whose members are precisely a, b, c and d and A has no other members except these members.

The set A = {1, 2, 3, 4, 5, 6, 7} described by listing method can be described by property method as:

$A = \{x \mid x \text{ is a positive integer and } x < 8\}$

Here, we have used the fact that every member of A is a positive integer less than 8 and every positive integer less than 8 is a member of A.

## The Empty Set

A set which does not contain any element is called the empty set or null set. The empty set is denoted by $\emptyset$. For example: The set of all prime numbers between 43 and 47 is an empty set.

# Subsets and Equal Sets

## Subsets

For sets A and B, if every member of A is also a member of B, then A is called a subset of B. This fact is shown symbolically as $A \subset B$.

Thus, if $\forall x,\ x \in A \Rightarrow x \in B$, then $A \subset B$. If $A \subset B$ then it can be said that B is a superset of A.

For example, if A = {2, 4, 6, 8} and B = {1, 2, 3, 4}, then there are members 6 and 8 in A which are not in B. Hence, A is not a subset of B.

Thus, $\exists x$ *such that* $x \in A$ and $x \notin B$

When $A \not\subset B$, there is an element $x$ in A which is not in B.

## Result

Empty set is a subset of every set i.e. $\emptyset \subset A$.

The subsets of A other than A and $\emptyset$ if they exist, are called proper subsets of A. For example, if B = {1, 4} and A = {1, 2, 3, 4, 5}, then B is a proper subset of A.

If B is a proper subset of A, then every member of B must be a member of A but every member of A is not a member of B.

The subsets of A = {0} are $\emptyset$ and {0}. The number of elements in A is 1 and the number of subsets of A is $2^1$.

The subsets of B = {-1, 1} are $\emptyset$, {-1}, {1} and {-1, 1}. The number of elements in B is 2 and the number of subsets of B is $2^2$.

The subsets of C = {1, 2, 3} are $\emptyset$, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3} and {1, 2, 3}. The number of elements in C is 3 and the number of subsets of C is $2^3$. Thus, if the number of elements in A is n, the number of its subsets is $2^n$.

**An Important Result**

If $A \subset B$ and $B \subset C$, then $A \subset C$

Here, $A \subset B$ and $B \subset C$

$\therefore A \subset C$

**Equal Sets**

If the sets A and B have exactly the same elements, then A and B are called equal sets and the same is written as A = B

For example if A = {2, 3, 4, 5}, $B = \{x \in N \mid 1 < x < 6\}$, then A = B. Also, $A \subset B$ and $B \subset A$

Thus, $A \subset B$ and $B \subset A \Rightarrow A = B$

# Types of Sets

**Objectives**

In this session we will understand the concept of cardinality of a set, types of sets and Equivalent sets.

We have understood the concept of set in our earlier session. Let us recapitulate. A set is a collection of well-defined, distinct objects.

We observe that there are some cars and motorcycles parked in a parking lot. If we consider only cars we can say that we have a set K a set of cars. Now, we proceed to learn other terms associated with a set.

In the set K of cars, there are 8 cars. So, the number of elements in this set is 8. Let us have another set $A = \{x \mid x \in N \text{ and } 6 < x < 14\}$

This set can also be written in Roster form as A = {7, 8, 9, 10, 11, 12, 13}. The set A has 7 elements.

The number of elements in a set is called the cardinal number of the set or Cardinality of the set and is written as n(A).

Thus, we have n(A) = 7; n(K) = 8

Now, we consider a few other sets to understand other types of sets.

B = Set of Red cars in K

P = {4}

$G = \{x \mid x \in Z \text{ and } -7 < x < -5\}$

= {-6}

We observe that all these sets have only one element in them. This leads us to the concept of a singleton set.

A set having only one element in it is called a Singleton Set. Let us observe a few sets.

K = set of cars

$F = \{2n - 1 / n \in N \text{ and } n < 14\}$

$F = \{1, 3, 5, 7, \ldots, 25\}$

Count the number of elements in these sets. We notice that the set K has 8 elements and the set F has 13 elements. Both the sets have a countable number of elements in them. Such sets are called finite sets.

A set having countable number of elements in it is called a finite set. Consider set of natural numbers N = {1, 2, 3, 4, 5,…} & set of integers Z = {…, -3, -2, -1, 0, 1, 2, 3, …}. These sets have uncountable elements in them; such sets are called infinite sets. A set with uncountable or infinite number of elements in it is called an infinite set.

Let us consider two sets A and B defined as A = {7, 8, 9, 10, 11, 12, 13} and B = {a, b, c, p, q, r, w}. We notice that cardinal number of set A and set B is the same. Such sets are called equivalent sets. The sets having the same number of elements in them are called Equivalent Sets. We express this correspondence as $A \sim B$.

**Summary**

In this session we have learnt about the

- Cardinal number: The number of elements in a set
- Singleton Set: The set having only one element
- Finite Set: The set with countable number of elements
- Infinite Sets: Sets having uncountable number of elements
- Equivalent Sets: The sets which have the same cardinal number.